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Batalin-Tyutin Quantization of the Self-Dual Massive Theory in Three Dimensions

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ABSTRACT

We quantize the self-dual massive theory by using the Batalin-Tyutin Hamiltonian method, which systematically embeds second class constraint system into first class one in the extended phase space by introducing the new fields. Through this analysis we obtain simultaneously the Stückelberg scalar term related to the explicit gauge-breaking effect and the new type of Wess-Zumino action related to the Chern-Simons term.

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1 Introduction

The Dirac method has been widely used in the Hamiltonian formalism [1] to quantize the second class constraint system. However, since the resulting Dirac brackets are generally field-dependent and nonlocal, and have a serious ordering problem between field operators, these are under unfavorable circumstances in finding canonically conjugate pairs. On the other hand, the quantizations of the first class constraint systems [2,3] have been well appreciated in a gauge invariant manner preserving Becci-Rouet-Stora-Tyutin (BRST) symmetry [4,5]. If the second class constraint system can be converted into first class one in an extended phase space, we do not need to define Dirac brackets and then the remaining quantization program follows the method of Ref. [2-5]. This procedure has been extensively studied by Batalin, Fradkin, and Tyutin [6,7] in the canonical formalism, and applied to various models [8-10] obtaining the Wess-Zumino (WZ) action [11,12].

Recently, Banerjee [13] has applied the Batalin-Tyutin (BT) Hamiltonian method [7] to the second class constraint system of the abelian Chern-Simons (CS) field theory [14-16], which yields first class constraint algebra in an extended phase space by introducing the new fields. As a result, he has obtained the new type of an abelian WZ action, which cannot be obtained in the usual path-integral framework. Very recently, we have quantized the nonabelian case, which yields the weakly involutive first class system originating from the second class one, by generalizing this BT formalism [17]. As shown in these works, the nature of second class constraint algebra originates from the symplectic structure of CS term, not due to the local gauge symmetry breaking. Banerjee, and Ghosh [18] have also considered a massive Maxwell theory, which has the explicit gauge-breaking term, in the BT approach. As a result, the extra field in this approach has identified with the Stückelberg scalar. There are some other interesting examples in this approach [19].

In the present paper, we shall apply the BT Hamiltonian method [7] to the self-dual massive theory [20] revealing both the Stückelberg effect [21] and CS effect [13,17] by using the BT quantization [7,18]. In section 2, since the BT formalism has developed quite recently, we first recapitulate this formalism by explicitly analyzing the pure CS

theory. In section 3, we apply the results discussed in section 2 to the well-known self-dual massive theory including the CS term in three dimensions, which is gauge non-invariant. By identifying the new fields ρ and λ with the Stückelberg scalar and WZ scalar, respectively, we obtain simultaneously the Stückelberg scalar term related to the explicit gauge-breaking mass term and the new type of WZ action, which also includes the Stückelberg scalar in order to maintain the gauge invariance related to the CS term.

2 The BT Formalism - The Pure CS Model

Now, we first recapitulate the BT formalism by analyzing the pure abelian CS model

$$S = \int d^3x [-\frac{m}{2}\epsilon_{\mu\nu\rho}B^\mu\partial^\nu B^\rho]. \tag{1}$$

Since this action is invariant up to the total divergence under the gauge transformation $\delta B^\mu = \partial^\mu\Lambda$, this action has a different origin of the second class constraint from the well-known massive Maxwell theory [18], which is due to the explicit gauge symmetry breaking term in the action. The origin of the second class constraints is due to the symplectic structure of the CS model.

Following the usual Dirac's standard procedure [1], we find that there are three primary constraints

$$\begin{aligned}\Omega_0 &\equiv \pi_0 \approx 0, \\ \Omega_i &\equiv \pi_i + \frac{1}{2}m\epsilon_{ij}B^j \approx 0 \quad (i = 1, 2),\end{aligned} \tag{2}$$

and one secondary constraint,

$$\omega_3 \equiv -m\epsilon_{ij}\partial^i B^j \approx 0, \tag{3}$$

obtained by conserving Ω_0 with the total Hamiltonian,

$$H_T = H_c + \int d^2x [u^0\Omega_0 + u^i\Omega_i], \tag{4}$$

where H_c is the canonical Hamiltonian,

$$H_c = \int d^2x \; [m\epsilon_{ij} B^0 \partial^i B^j], \tag{5}$$

and we denote $x = (t, \vec{x})$ and two-space vector $\vec{x} = (x^1, x^2)$ and $\epsilon_{12} = \epsilon^{12} = 1$ and Lagrange multipliers u^0, u^i. No further constraints are generated via this iterative procedure. We find that all rest constraints except $\Omega_0 = \pi_0 \approx 0$ are superficially second class constraints. However, in order to extract out the true second class constraints, it is essential to redefine ω_3 by using Ω_1 and Ω_2 as follows

$$\begin{aligned} \Omega_3 &\equiv \omega_3 + \partial^i \Omega_i \\ &= \partial^i \pi_i - \frac{1}{2} m \epsilon_{ij} \partial^i B^j. \end{aligned} \tag{6}$$

Then, Ω_0, Ω_3 form the first class algebra, while Ω_1, Ω_2 form the second class algebra as follows

$$\begin{aligned} \Delta_{ij}(x,y) &\equiv \{\Omega_i(x), \Omega_j(y)\} \\ &= \begin{pmatrix} 0 & m \\ -m & 0 \end{pmatrix} \delta^2(x-y); \quad i,j = 1,2. \end{aligned} \tag{7}$$

In order to convert this system into first class one, the first objective is to transform Ω_i into the first class by extending the phase space. Following the BT approach [7], we introduce new auxiliary fields Φ^i to convert the second class constraint Ω_i into the first class one in the extended phase space, and assume that the Poisson algebra of the new fields is given by

$$\{\Phi^i(x), \Phi^j(y)\} = \omega^{ij}(x,y), \tag{8}$$

where ω^{ij} is an antisymmetric matrix. Then, the modified constraint in the extended phase space is given by

$$\tilde{\Omega}_i(\pi_\mu, B^\mu, \Phi^i) = \Omega_i + \sum_{n=1}^{\infty} \Omega_i^{(n)}; \qquad \Omega_i^{(n)} \sim (\Phi^i)^n, \tag{9}$$

satisfying the boundary condition, $\tilde{\Omega}_i(\pi_\mu, B^\mu, \Phi^i = 0) = \Omega_i$. The first order correction term in the infinite series [7] is given by

$$\Omega_i^{(1)}(x) = \int d^2y X_{ij}(x,y) \Phi^j(y), \tag{10}$$

and the first class constraint algebra of $\tilde{\Omega}_i$ requires the condition as follows

$$\triangle_{ij}(x,y) + \int d^2w\ d^2z\ X_{ik}(x,w)\omega^{kl}(w,z)X_{lj}(z,y) = 0. \tag{11}$$

As was emphasized in Ref. [13,17], there is a natural arbitrariness in choosing ω^{ij} and X_{ij} from Eq.(8) and Eq.(10), which corresponds to the canonical transformation in the extended phase space [6,7]. We take the simple solutions as

$$\begin{aligned} \omega^{ij}(x,y) &= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \delta^2(x-y), \\ X_{ij}(x,y) &= \begin{pmatrix} \sqrt{m} & 0 \\ 0 & \sqrt{m} \end{pmatrix} \delta^2(x-y), \end{aligned} \tag{12}$$

which are compatible with Eq.(11) as it should be. Using Eqs.(9), (10) and (12), the new set of constraints is found to be

$$\tilde{\Omega}_i = \pi_i + \frac{1}{2}m\epsilon_{ij}B^j + \sqrt{m}\Phi^i, \qquad (i=1,2), \tag{13}$$

which are strongly involutive,

$$\{\tilde{\Omega}_\alpha, \tilde{\Omega}_\beta\} = 0 \qquad (\alpha,\beta = 0,1,2,3) \tag{14}$$

with $\tilde{\Omega}_0 \equiv \Omega_0$ and $\tilde{\Omega}_3 \equiv \Omega_3$. As a result, we have all first class constraints in the extended phase space by applying the BT formalism systematically. Observe further that only $\Omega_i^{(1)}$ contributes in the series (9) defining the first class constraint. All higher order terms given by Eq.(9) vanish as a consequence of the choice Eq.(12).

Next, we derive the corresponding involutive Hamiltonian in the extended phase space. It is given by the infinite series [7],

$$\tilde{H} = H_c + \sum_{n=1}^{\infty} H^{(n)}; \qquad H^{(n)} \sim (\Phi^i)^n, \tag{15}$$

satisfying the initial condition, $\tilde{H}(\pi_\mu, B^\mu, \Phi^i = 0) = H_c$. The general solution [7] for the involution of $\tilde{H}$ is given by

$$H^{(n)} = -\frac{1}{n}\int d^2xd^2yd^2z\ \Phi^i(x)\omega_{ij}(x,y)X^{jk}(y,z)G_k^{(n-1)}(z), \quad (n \geq 1), \tag{16}$$

where the generating functions $G_k^{(n)}$ are given by

$$\begin{aligned} G_i^{(0)} &= \{\Omega_i^{(0)}, H_c\}, \\ G_i^{(n)} &= \{\Omega_i^{(0)}, H^{(n)}\}_{\mathcal{O}} + \{\Omega_i^{(1)}, H^{(n-1)}\}_{\mathcal{O}} \quad (n \geq 1), \end{aligned} \tag{17}$$

where the symbol $\mathcal{O}$ in Eq.(17) represents that the Poisson brackets are calculated among the original variables, i.e., $\mathcal{O} = (\pi_\mu, B^\mu)$. Here, ω_{ij} and X^{ij} are the inverse matrices of ω^{ij} and X_{ij} respectively. Explicit calculations yield,

$$G_i^{(0)} = -m\epsilon_{ij}\partial^j B^0, \tag{18}$$

which is substituted in Eq.(16) to obtain $H^{(1)}$,

$$H^{(1)} = \int d^2x[-\sqrt{m}(\partial_i\Phi^i)B^0]. \tag{19}$$

In this case, there are no further iterative Hamiltonians. Thus, the total corresponding canonical Hamiltonian is

$$\tilde{H} = H_c + H^{(1)}, \tag{20}$$

which is involutive with the first class constraints,

$$\begin{aligned} \{\tilde{\Omega}_i, \tilde{H}\} &= 0, \qquad (i = 1, 2, 3), \\ \{\tilde{\Omega}_0, \tilde{H}\} &= \tilde{\Omega}_3. \end{aligned} \tag{21}$$

This completes the operatorial conversion of the original second class system with Hamiltonian H_c and constraints Ω_i into first class with the involutive Hamiltonian $\tilde{H}$ and constraints $\tilde{\Omega}_i$. Note that this Hamiltonian naturally generates the first class Gauss' law constraint $\tilde{\Omega}_3$ from the time evolution of $\tilde{\Omega}_0$.

Let us identify the new variables Φ^i as a canonically conjugate pair (λ, π_λ) in the Hamiltonian formalism,

$$\begin{aligned} \Phi^1 &\rightarrow \frac{1}{\sqrt{m}}\pi_\lambda, \\ \Phi^2 &\rightarrow \sqrt{m}\lambda \end{aligned} \tag{22}$$

satisfying Eqs.(8) and (12). Then, the starting phase space partition function is given by the Faddeev formula [3,22] as follows

$$Z = \int \mathcal{D}B^\mu \mathcal{D}\pi_\mu \mathcal{D}\lambda \mathcal{D}\pi_\lambda \prod_{\alpha,\beta=0}^{3} \delta(\tilde{\Omega}_\alpha)\delta(\Gamma_\beta) det \mid \{\tilde{\Omega}_\alpha, \Gamma_\beta\} \mid e^{iS}, \tag{23}$$

where

$$S = \int d^3x \left(\pi_\mu \dot{B}^\mu + \pi_\lambda \dot{\lambda} - \tilde{\mathcal{H}}\right), \tag{24}$$

with Hamiltonian density $\tilde{\mathcal{H}}$ corresponding to Hamiltonian $\tilde{H}$, which is now expressed in terms of (λ, π_λ) instead of Φ^i. The gauge fixing conditions Γ_i are chosen so that the determinant occurring in the functional measure is nonvanishing. Moreover, Γ_i may be assumed to be independent of the momenta so that these are considered as Faddeev-Popov type gauge conditions [23].

We now perform the momentum integrations to obtain the configuration space partition function. The π_0, π_1, and π_2 integrations are trivially performed by exploiting the delta function $\delta(\tilde{\Omega}_0) = \delta(\pi_0)$, $\delta(\tilde{\Omega}_1) = \delta(\pi_1 + \frac{m}{2}B^2 + \pi_\lambda)$, and $\delta(\tilde{\Omega}_2) = \delta(\pi_2 - \frac{m}{2}B^1 + m\lambda)$, respectively. After exponentiating the remaining delta function $\delta(\tilde{\Omega}_3) = \delta(-m\epsilon_{ij}\partial^i B^j + \partial_1 \pi_\lambda + m\partial_2\lambda)$ with Fourier variable ξ and transforming $B^0 \rightarrow B^0 + \xi$, we finally obtain the action as follows

$$S = \int d^3x [-\frac{1}{2}m\epsilon_{\mu\nu\rho}B^\mu\partial^\nu B^\rho + m\lambda F_{02}], \tag{25}$$

where $F_{02} = \partial_0 B_2 - \partial_2 B_0$, and the corresponding Liouville measure just comprises the configuration space variables as follows

$$[\mathcal{D}\mu] = \mathcal{D}B^\mu \mathcal{D}\lambda \mathcal{D}\xi \delta(F_{01} + \dot{\lambda}) \prod_{\beta=0}^{3} \{\delta(\Gamma_\beta[B^0 + \xi, B^i, \lambda])\} det \mid \{\tilde{\Omega}_\alpha, \Gamma_\beta\} \mid, \tag{26}$$

where $\delta(F_{01} + \dot{\lambda})$ is expressed by $\int \mathcal{D}\pi_\lambda e^{-i\int d^3x\ (F_{01}+\dot{\lambda})\pi_\lambda}$. This action is invariant up to the total divergence under the gauge transformations as $\delta B_\mu = \partial_\mu \Lambda$ and $\delta\lambda = 0$. It is easily checked for consistency that starting from the Lagrangian (25) with a factor $\delta(F_{01} + \dot{\lambda})$ in the measure part, one can exactly reproduce the set of all first class constraints $\tilde{\Omega}_\alpha$ and the involutive Hamiltonian (20). Note that we will show in the next section that the above δ-function, which is remaining in the measure part for the case of the pure CS theory, will be disappeared for the case of the non-pure CS theories [13,17] like the self-dual massive theory [20].

3 The Self-Dual Massive Model

We consider the Abelian self-dual massive model [20]

$$S_{SD} = \int d^3x\ [\frac{1}{2}m^2 B^\mu B_\mu - \frac{1}{2}m\epsilon_{\mu\nu\rho}B^\mu\partial^\nu B^\rho], \tag{27}$$

by using the useful results discussed in the previous section. Note that this action has an explicit mass term, which breaks the gauge symmetry as the case of the Proca model, and also the CS term, which has a different origin of the second class constraint system. Consequently, this action represents a second class constraint system, which can be easily confirmed by the standard constraint analysis. There are three primary constraints,

$$\begin{aligned} \Omega_0 &\equiv \pi_0 \approx 0, \\ \Omega_i &\equiv \pi_i + \frac{1}{2} m \epsilon_{ij} B^j \approx 0, \qquad (i = 1, 2), \end{aligned} \tag{28}$$

and one secondary constraint,

$$\omega_3 \equiv m^2 B^0 - m \epsilon_{ij} \partial^i B^j \approx 0, \tag{29}$$

which is obtained by conserving Ω_0 with the total Hamiltonian,

$$H_T = H_c + \int d^2x [u^0 \Omega_0 + u^i \Omega_i], \tag{30}$$

where H_c is the canonical Hamiltonian,

$$H_c = \int d^2x \left[\frac{1}{2} m^2 \{ (B^i)^2 - (B^0)^2 \} + m \epsilon_{ij} B^0 \partial^i B^j \right], \tag{31}$$

and u^0 and u^i are Lagrange multipliers. No further constraints are generated via this iterative procedure. We find that all constraints are fully second class constraints. It is, however, essential to redefine ω_3 by using Ω_1 and Ω_2 as follows

$$\begin{aligned} \Omega_3 &\equiv \omega_3 + \partial^i \Omega_i \\ &= \partial^i \pi_i - \frac{1}{2} m \epsilon_{ij} \partial^i B^j + m^2 B^0, \end{aligned} \tag{32}$$

although the redefined constraints are still completely second class in contrast to the case of the pure CS theory. Otherwise, one will have a complicated constraint algebra including the derivative terms, which is difficult to handle. Then, the constraint algebra is given by

$$\begin{aligned} \Delta_{\alpha\beta}(x, y) &\equiv \{ \Omega_\alpha(x), \Omega_\beta(y) \} \\ &= \begin{pmatrix} 0 & 0 & 0 & -m^2 \\ 0 & 0 & m & 0 \\ 0 & -m & 0 & 0 \\ m^2 & 0 & 0 & 0 \end{pmatrix} \delta^2(x - y); \alpha, \beta = 0, 1, 2, 3, \end{aligned} \tag{33}$$

which reveals the simple second class nature of the constraints $\Omega_\alpha(x)$.

In order to convert this system into the first class one, the first objective is to transform Ω_α into the first class by extending the phase space. Following the BT approach [7], we introduce the matrix (8) as follows

$$\omega^{\alpha\beta}(x,y) = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \delta^2(x-y). \tag{34}$$

Then the other matrix $X_{\alpha\beta}$ in Eq.(10) is obtained by solving Eq.(11) with $\Delta_{\alpha\beta}$ given by Eq.(33),

$$X_{\alpha\beta}(x,y) = \begin{pmatrix} m & 0 & 0 & 0 \\ 0 & \sqrt{m} & 0 & 0 \\ 0 & 0 & \sqrt{m} & 0 \\ 0 & 0 & 0 & m \end{pmatrix} \delta^2(x-y). \tag{35}$$

There is an arbitrariness in choosing $\omega^{\alpha\beta}$, which would naturally be manifested in Eq.(34). This just corresponds to the canonical transformations in the extended phase space. However, as has also been evidenced in other calculations [13,17], this choice of Eqs.(34) and (35) brings about remarkable algebraic simplifications.

Using Eqs.(9), (10) and (35), the new set of constraints is found to be

$$\begin{aligned} \tilde{\Omega}_0 &= \pi_0 + m\Phi^0, \\ \tilde{\Omega}_i &= \pi_i + \frac{1}{2}m\epsilon_{ij}B^j + \sqrt{m}\Phi^i, \qquad (i=1,2), \\ \tilde{\Omega}_3 &= \partial^i\pi_i - \frac{1}{2}m\epsilon_{ij}\partial^iB^j + m^2B^0 + m\Phi^3, \end{aligned} \tag{36}$$

which are strongly involutive,

$$\{\tilde{\Omega}_\alpha, \tilde{\Omega}_\beta\} = 0. \tag{37}$$

Recall the Φ^μ are the new variables satisfying the algebra (8) with $\omega^{\alpha\beta}$ given by Eq.(34).

The next step is to obtain the involutive Hamiltonian. The generating functional $G_\alpha^{(n)}$ is obtained from Eq.(17). It is noteworthy that there are only two terms Ω_α and $\tilde{\Omega}_\alpha^{(1)}$ in the expansion (36) due to the intuitive choice (34) and (35). Explicit calculations yield,

$$G_0^{(0)} = m^2B^0 - m\epsilon_{ij}\partial^iB^j,$$

$$G_i^{(0)} = -m^2 B^i - m\epsilon_{ij}\partial^j B^0,$$
$$G_3^{(0)} = m^2\partial_i B^i, \qquad (38)$$

which are substituted in Eq.(16) to obtain $H^{(1)}$,

$$H^{(1)} = \int d^2x \left[m\Phi^0\partial_i B^i + m\sqrt{m}\epsilon_{ij}\Phi^i B^j + \sqrt{m}\Phi^i\partial_i B^0 - \Phi^3(mB^0 - \epsilon_{ij}\partial^i B^j)\right]. \qquad (39)$$

This is inserted back in Eq.(17) to deduce $G_\alpha^{(1)}$ as follows

$$G_0^{(1)} = \sqrt{m}\partial_i\Phi^i + m\Phi^3,$$
$$G_i^{(1)} = m\partial_i\Phi^0 + m\sqrt{m}\epsilon_{ij}\Phi^j - \epsilon_{ij}\partial^j\Phi^3,$$
$$G_3^{(1)} = m\partial_i\partial^i\Phi^0 + m\sqrt{m}\epsilon_{ij}\partial^i\Phi^j, \qquad (40)$$

which then yield $H^{(2)}$ from Eq.(16),

$$H^{(2)} = \int d^2x \left[-\frac{1}{2}\partial_i\Phi^0\partial^i\Phi^0 + \sqrt{m}\Phi^0\epsilon_{ij}\partial^i\Phi^j + \frac{1}{2}m\Phi^i\Phi^i - (\frac{1}{\sqrt{m}}\partial_i\Phi^i + \frac{1}{2}\Phi^3)\Phi^3\right]. \qquad (41)$$

Since $G_\alpha^{(n)} = 0 \quad (n \geq 2)$, the final expression for the desired involutive Hamiltonian after the $n = 2$ finite truncations is given by

$$\tilde{H} = H_c + H^{(1)} + H^{(2)}, \qquad (42)$$

which, by construction, is involutive,

$$\{\tilde{\Omega}_\alpha, \tilde{H}\} = 0. \qquad (43)$$

This completes the operatorial conversion of the original second class system with Hamiltonian H_c and constraints Ω_α into first class with Hamiltonian $\tilde{H}$ and constraints $\tilde{\Omega}_\alpha$.

Before performing the momentum integrations to obtain the partition function in the configuration space, it seems appropriate to comment on the involutive Hamiltonian. If we directly use the above Hamiltonian, we will finally obtain the non-local action corresponding to this Hamiltonian due to the existence of $\frac{1}{\sqrt{m}}\Phi^3\partial_1\Phi^1$–term in the action when we carry out the functional integration over Φ^1 or Φ^3 later. Furthermore, if we use the above Hamiltonian, we can not also naturally generate the

first class Gauss' law constraint $\tilde{\Omega}_3$ from the time evolution of the primary constraint $\tilde{\Omega}_0$, which is the first class. Therefore, in order to avoid these problems, we use the equivalent first class Hamiltonian without any loss of generality, which differs from the involutive Hamiltonian (42) by adding a term proportional to the first class constraint $\tilde{\Omega}_3$ as follows

$$\tilde{H}' = \tilde{H} + \frac{1}{m}\Phi^3\tilde{\Omega}_3. \tag{44}$$

Then, this Hamiltonian $\tilde{H}'$ consistently generates the Gauss' law constraint such that $\{\tilde{\Omega}_0, \tilde{H}'\} = \tilde{\Omega}_3$. Note that non-locality may also be avoided by changing the order of performing the momentum integrals. But, in this case, one can not directly reproduce the original theory by fixing the unitary gauge as well as the Gauss' law constraint. Furthermore, when we act this modified Hamiltonian on physical states, the difference with $\tilde{H}$ is trivial because such states are annihilated by the first class constraints. Similarly, the equations of motion for observable (*i.e.* gauge invariant variables) will also be unaffected by this difference since $\tilde{\Omega}_3$ can be regarded as the generator of the gauge transformations.

We now unravel the correspondence of the Hamiltonian approach including both the Stückelberg effect and the CS effect. The first step is to identify the new variables Φ^μ as canonically conjugate pairs in the Hamiltonian formalism,

$$\begin{aligned}
\Phi^0 &\rightarrow m\rho, \\
\Phi^1 &\rightarrow \frac{1}{\sqrt{m}}\pi_\lambda, \\
\Phi^2 &\rightarrow \sqrt{m}\lambda, \\
\Phi^3 &\rightarrow \frac{1}{m}\pi_\rho,
\end{aligned} \tag{45}$$

satisfying Eqs.(8), (34) and (35). The starting phase space partition function is then given by the Faddeev formula,

$$Z = \int \mathcal{D}B^\mu \mathcal{D}\pi_\mu \mathcal{D}\lambda \mathcal{D}\pi_\lambda \mathcal{D}\rho \mathcal{D}\pi_\rho \prod_{\alpha,\beta=0}^{3} \delta(\tilde{\Omega}_\alpha)\delta(\Gamma_\beta) det \mid \{\tilde{\Omega}_\alpha, \Gamma_\beta\} \mid e^{iS'}, \tag{46}$$

where

$$S' = \int d^3x \left(\pi_\mu \dot{B}^\mu + \pi_\lambda \dot{\lambda} + \pi_\rho \dot{\rho} - \tilde{\mathcal{H}}'\right) \tag{47}$$

with the Hamiltonian density $\tilde{\mathcal{H}}'$ corresponding to $\tilde{H}'$, which is now expressed in terms of $(\rho, \pi_\rho, \lambda, \pi_\lambda)$ instead of Φ^μ. As in the previous section, the gauge fixing conditions Γ_α may be assumed to be independent of the momenta so that these are considered as Faddeev-Popov type gauge conditions.

Next, we perform the momentum integrations to obtain the configuration space partition function. The π_0, π_1, and π_2 integrations are trivially performed by exploiting the delta functions $\delta(\tilde{\Omega}_0) = \delta(\pi_0 + m^2\rho)$, $\delta(\tilde{\Omega}_1) = \delta(\pi_1 + \frac{m}{2}B^2 + \pi_\lambda)$, and $\delta(\tilde{\Omega}_2) = \delta(\pi_2 - \frac{m}{2}B^1 + m\lambda)$, respectively. After exponentiating the remaining delta function $\delta(\tilde{\Omega}_3) = \delta(-m\epsilon_{ij}\partial^i B^j + \partial_1\pi_\lambda + m\partial_2\lambda + m^2 B^0 + \pi_\rho)$ with Fourier variable ξ as $\delta(\tilde{\Omega}_3) = \int \mathcal{D}\xi e^{-i\int d^3x\xi\tilde{\Omega}_3}$ and transforming $B^0 \rightarrow B^0 + \xi$, we obtain the action as follows

$$
\begin{aligned}
S &= \int d^3x\ \{\frac{1}{2}m^2 B^\mu B_\mu - \frac{1}{2}m\epsilon_{\mu\nu\rho}B^\mu\partial^\nu B^\rho \\
&+ \rho[-m^2(\dot{B}^0 + \dot{\xi}) - m^2\partial_i B^i - \frac{1}{2}m^2\partial_i\partial^i\rho + m^2\partial_1\lambda - m\partial_2\pi_\lambda] \\
&+ \pi_\rho[\dot{\rho} - \frac{1}{2m^2}\pi_\rho - \xi] + \lambda[-m\dot{B}^2 + m^2 B^1 - m\partial_2 B^0 - \frac{1}{2}m^2\lambda] \\
&+ \pi_\lambda[\dot{\lambda} - \dot{B}^1 - mB^2 + \partial^1 B^0 - \frac{1}{2}\pi_\lambda] - \frac{1}{2}m^2\xi^2\},
\end{aligned} \tag{48}
$$

and the corresponding measure is given by

$$
[\mathcal{D}\mu] = \mathcal{D}B^\mu\mathcal{D}\lambda\mathcal{D}\pi_\lambda\mathcal{D}\rho\mathcal{D}\pi_\rho\mathcal{D}\xi\prod_{\beta=0}^{3}\{\delta(\Gamma_\beta[B^0+\xi, B^i, \lambda, \rho])\}det \mid \{\tilde{\Omega}_\alpha, \Gamma_\beta\} \mid, \tag{49}
$$

where $B^0 \rightarrow B^0 + \xi$ transformation is naturally understood in the gauge fixing condition Γ_β.

Note that the original theory is easily reproduced in one line, if we choose the unitary gauge

$$
\Gamma_0 = \rho, \quad \Gamma_1 = \pi_\lambda, \quad \Gamma_2 = \lambda, \quad \Gamma_3 = \pi_\rho, \tag{50}
$$

and integrate over ξ. Then, one can easily realize that the new fields Φ^μ are nothing but the gauge degrees of freedom, which can be removed by utilizing the gauge symmetry.

Now, we perform the Gaussian integration over π_ρ. Then all ξ terms in the action are canceled out, and integrating over π_λ, the resultant action is finally obtained as follows

$$
S = S_{Stückelberg} + S_{WZ};
$$

$$
\begin{aligned}
S_{St\ddot{u}ckelberg} &= \int d^3x\{\frac{1}{2}m^2(B_\mu B^\mu + \partial_\mu\rho\partial^\mu\rho - 2\rho\partial_\mu B^\mu) - \frac{1}{2}m\epsilon_{\mu\nu\rho}B^\mu\partial^\nu B^\rho\}, \\
&= \int d^3x\{\frac{1}{2}m^2(B_\mu + \partial_\mu\rho)^2 - m^2\partial_\mu(\rho B^\mu) - \frac{1}{2}m\epsilon_{\mu\nu\rho}B^\mu\partial^\nu B^\rho\}, \\
S_{WZ} &= \int d^3x\{\frac{1}{2}[\dot{\lambda} + F_{01} + m(B_2 + \partial_2\rho)]^2 \\
&\qquad + m\lambda[F_{02} - m(B_1 + \partial_1\rho) - \frac{1}{2}m\lambda]\},
\end{aligned} \tag{51}
$$

where $F_{01} = \partial_0 B_1 - \partial_1 B_0$, $F_{02} = \partial_0 B_2 - \partial_2 B_0$, and the corresponding Liouville measure just comprises the configuration space variables as follows

$$
[\mathcal{D}\mu] = \mathcal{D}B^\mu\mathcal{D}\lambda\mathcal{D}\rho\mathcal{D}\xi\prod_{\beta=0}^{3}\{\delta(\Gamma_\beta[B^0+\xi, B^i, \lambda, \rho])\}det\mid\{\tilde{\Omega}_\alpha, \Gamma_\beta\}\mid. \tag{52}
$$

This action S is invariant up to the total divergence under the gauge transformations as $\delta B_\mu = \partial_\mu\Lambda$, $\delta\rho = -\Lambda$, and $\delta\lambda = 0$. Starting from the Lagrangian (51) with the boundary term, we can easily reproduce the same set of all first class constraints $\tilde{\Omega}_\alpha$, and the Hamiltonian such that

$$
\begin{aligned}
H = H_c &+ \int d^2x \ \ [\pi_\lambda\partial_1 B_0 - m\pi_\lambda + m^2\rho\partial_i B^i + m\lambda\partial_2 B_0 + m^2\lambda B_1 \\
&+ \frac{1}{2}\pi_\lambda^2 - m\pi_\lambda\partial_2\rho - \frac{1}{2}m^2\partial_i\rho\partial^i\rho + m^2\lambda\partial_1\rho + \frac{1}{2}m^2\lambda^2 + \frac{1}{2}\pi_\rho^2].
\end{aligned} \tag{53}
$$

Then, if we add a term proportional to the constraint $\tilde{\Omega}_3$, *i.e.*, $-\frac{1}{m^2}\pi_\rho\tilde{\Omega}_3$, which is trivial when acting on the physical Hilbert space, to the above Hamiltonian (53), we can obtain the original involutive Hamiltonian (42), which is canonically equivalent to Eq.(53). Furthermore, this difference is also trivial in the construction of the functional integral because the constraint $\tilde{\Omega}_3$ is strongly implemented by the delta function $\delta(\tilde{\Omega}_3)$ in Eq.(46). Therefore, we have shown that the constraints and Hamiltonian following from the Lagrangian (51) are effectively equivalent to the original Hamiltonian embedding. As results, through BT quantization procedure, we have found that the Stückelberg scalar ρ is naturally introduced in the mass term, and this ρ as well as the WZ scalar λ is also included in the new type of WZ action.

Note that the gauge invariance of $S_{\rm WZ}$ should be maintained because the second-class constraint structure related to CS term only comes from the symplectic structure. We also note that the Wess-Zumino action in Eq.(51) is gauge invariant in spite of the

lack of the manifest Lorentz invariance. On the other hand, since the unitary gauge (50) recovers the manifestly Lorentz invariant original action, the actual invariance is maintained from the fact that the final result for the partition function Z is independent of the gauge fixing conditions. The local gauge symmetry of the Wess-Zumino action naturally also survives in the configuration space. This means that the origin of S_{WZ} is irrelevant to the conventional gauge-variant Wess-Zumino like action [8,10,11], which cancels the local gauge anomaly of the second-class system. Interestingly the choice of $\rho = 0$ and λ=0 does not recover the original theory in the Faddeev-Popov type gauges. Finally, it seems appropriate to comment on the action (51). If we ignore the boundary term in the Lagrangian (51), we cannot directly obtain the involutive first class Hamiltonian as the case of the Proca theory explained in Ref. [18] because the boundary term plays the important role in this procedure.

In summary, we have recapitulated the Batalin-Tyutin method, which converts second class system into first class one, by analyzing the pure CS theory, which has the different origin of the second class structure. Then, we have applied this method to the Abelian self-dual massive theory including the CS term. As results, we have shown that if we ignore the boundary term in action (51), the direct connection with the usual Lagrangian embedding of Stückelberg can be made by explicitly evaluating the momentum integrals in the extended phase space partition function using Faddeev-Popov-like gauges and identifying an extra field ρ introduced in our Hamiltonian formalism with the conventional Stückelberg scalar. On the other hand, we have also obtained a new type of Wess-Zumino action containing the WZ scalar λ, which also includes the Stückelberg scalar ρ in order to maintain the gauge invariance of the S_{WZ} related to the CS effect in the action (51).

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